On some hybrid-types of Q balls in the gauge-mediated supersymmetry breaking

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We revisit the new-type of the Q ball (the gravity-mediation type of the Q ball in the gauge-mediation), in order to clarify its properties and correct some misunderstandings found in the recent literature. In addition, we investigate the feature of the other kind of the hybrid-type of the Q ball, which was considered in the context of the Q-ball capture by the neutron star.

I. INTRODUCTION

A Q ball is a nontopological soliton, the minimum energy configuration of the (complex) scalar field, whose existence is guaranteed by non-zero charge Q [1]. It appears naturally in supersymmetric theories [2, 3]. In particular in the gauge-mediated supersymmetry (SUSY) breaking, the Q ball is stable against the decay into fermions and other scalars for large enough charge Q, and can be the dark matter of the universe [2]. Such large Q balls are naturally produced in the early universe as byproducts of the Affleck-Dine mechanism for baryogenesis [2, 4, 5, 6]. They could be detectable and/or constrained by various experiments [6, 7, 8] and by considering astrophysically such as the capture by neutron stars [9, 10].

The properties of the Q ball is determined by the shape of the scalar potential. In the gauge-mediated SUSY breaking, the potential is flat beyond the messenger scale, and the mass grows as $M \propto Q^{3/4}$ [11]. As the field amplitude becomes large, the potential will be dominated by the effect of gravity-mediation, and the features of the Q ball change such as $M \propto Q$, for example. We called this kind of the Q ball the new-type Q ball [12]. As we mentioned in [12], the 'metamorphosis' of the Q ball should take place smoothly.

In this article, we revisit the properties of the new-type Q balls, with special attention to clarify the transition region between the gauge and gravity mediation. This is partly because we must correct some misunderstandings found in [13], where they claim the new-type Q ball disintegrates into the gauge-type ones.

In addition, we also investigate the features of the other kind of hybrid type of the Q ball considered in [10], where the gauge-type Q ball changes to the thin-wall-type Q ball[1] as it fattens in the interior of the neutron star. This happens since the field value inside the Q ball cannot grow further when the potential is lifted by nonrenormalizable operators at large field values.[2]

II. Q BALL SOLUTION

Let us first review the general properties of the Q ball in the scalar theory wtih a global $U(1)$ symmetry. In the context of SUSY Q balls, the charge is usually the baryon and/or lepton numbers. The energy and charge are given respectively by

$$E = \int d^3x \left[\partial_\mu\phi\partial^\mu\phi^* + V(\phi)\right], \tag{1}$$

$$Q = \frac{1}{i}\int d^3x(\dot\phi\phi^* - \phi\dot\phi^*), \tag{2}$$

where $V(\phi)$ is the potential. Since the Q ball is the energy minimum configuration of the scalar field with finite charge Q, using a lagrange multiplier ω, we can write the energy as [14]

$$\mathcal{E}_\omega = E + \omega\left[Q - \frac{1}{i}\int d^3x(\dot\phi\phi^* - \phi\dot\phi^*)\right]. \tag{3}$$

[1] In Ref. [10], it is called the 'curved direction' Q ball. In this article, we call it the 'thin-wall-type' Q ball because of its profile as shown in Fig. 2.

[2] This thin-wall-type Q ball is not created through the Affleck-Dine mechanism, since the scalar field does not feel spatial instabilities while it stays on the nonrenormalizable potential.

Energy minimum configuration is obtained when the solution is spherical symmetric and rotating: $\phi(x) = \varphi(r)e^{i\omega t}/\sqrt{2}$. Then, the energy is rewritten as

$$\mathcal{E}_\omega = 4\pi \int_0^\infty drr^2 \left[\frac{1}{2}\left(\frac{d\varphi}{dr}\right)^2 + V(\varphi) - \frac{1}{2}\omega^2\varphi^2 \right] + \omega Q. \tag{4}$$

In order to obtain the energy minimum solution, we just have to solve the equation

$$\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr} + \left[\omega^2\varphi - \frac{dV}{d\varphi}\right] = 0, \tag{5}$$

with boundary conditions $\varphi(\infty) = 0$ and $\varphi'(0) = 0$.

In the next sections, we apply the above argument and solve the equation (5) numerically for two kinds of hybrid-type Q balls: One is the gauge-type and new-type Q balls, and the other is the gauge-type and thin-wall-type Q balls.

III. NEW-TYPE Q BALLS

The potential is written as [12]

$$V(\phi) = m_\phi^4 \log\left(1 + \frac{|\phi|^2}{m_\phi^2}\right) + m_{3/2}^2|\phi|^2\left(1 + K\log\frac{|\phi|^2}{M_*^2}\right), \tag{6}$$

where the first (second) term comes from the gauge- (gravity-)mediation effect. Here m_ϕ is the scalar mass in the vacuum, $m_{3/2}$ the gravitino mass, $K < 0$ the coefficient of the one-loop effect, and M_* a renormalization scale. When the each term of the potential dominates, the properties of each type of the Q ball are well known. For the gauge-type Q ball, the energy E, the size R, the rotation speed ω, and the field value at the center φ_c are given by [2, 6, 11]

$$\begin{aligned} &E \sim m_\phi Q^{3/4}, \\ &R \sim \omega^{-1} \sim m_\phi^{-1}Q^{1/4}, \\ &\varphi_c \sim m_\phi Q^{1/4}, \end{aligned} \tag{7}$$

while for the new-type Q ball [3, 12],

$$\begin{aligned} &E \sim m_{3/2}Q, \\ &R \sim |K|^{-1/2}m_{3/2}^{-1}, \\ &\omega \sim m_{3/2}, \\ &\varphi_c \sim m_{3/2}Q^{1/2}. \end{aligned} \tag{8}$$

To look also for the transition region, we solve numerically Eq.(5) for the potential (6). In Fig. 1, the energy, size, the field value, and energy per charge as a function of the charge are shown. In these figures, we set $m_{3/2}/m_\phi = 10^{-5}$, $K = -0.01$, $M_*/m_\phi = 10^{10}$. One can see the features of both gauge-type and new-type Q balls, *i.e.*, Eqs.(7) and (8), are reproduced in Fig. 1. Moreover, those parameters are smoothly connected in the transition region, as it should be. In particular, the energy per charge E/Q always decreases as the charge Q increases, which implies that the larger Q ball is energetically favored. This shows that the new-type Q ball is stable against disintegration into smaller gauge-type Q balls, so that it can be the dark matter of the universe, contrary to the claim in Ref. [13].

IV. THIN-WALL-TYPE Q BALLS

Now let us consider the following potential,

$$V = m_\phi^4 \log\left(1 + \frac{|\phi|^2}{m_\phi^2}\right) + \frac{\lambda^2|\phi|^{2(n-1)}}{M_P^{2(n-3)}}. \tag{9}$$

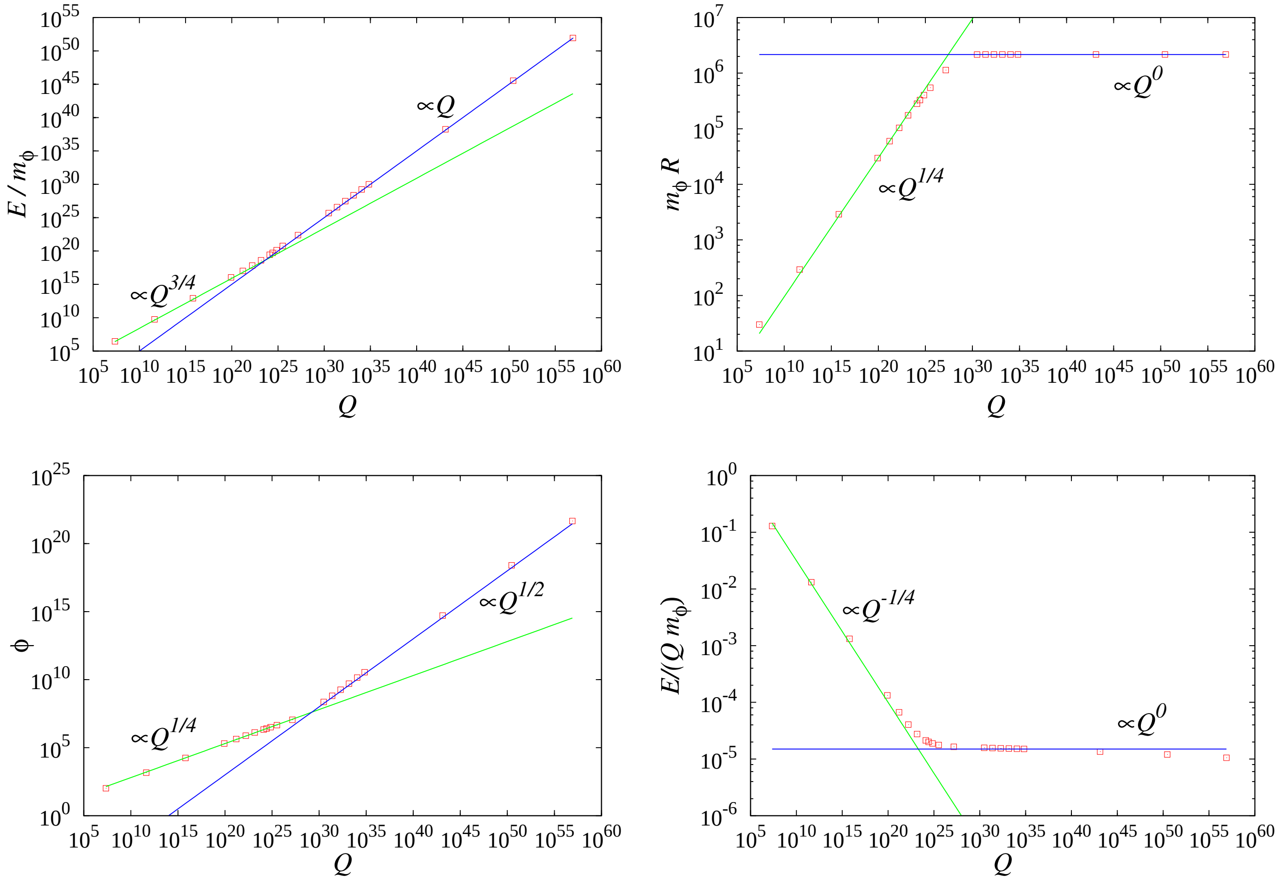

FIG. 1: Energy, size, field value at the center, and energy per charge of the Q balls. Green and blue lines show the Q-dependence estimated analytically for the gauge-type and new-type Q balls, respectively.

Although the thin-wall-type Q ball is not created in the early universe, it could be formed through charge accumulation from the gauge-type Q ball when the latter is swallowed by the neutron star [10]. The properties of the thin-wall-type Q ball are also well known as [1, 10]

$$\begin{aligned}
&E \sim \mu Q, \\
&R \sim \left(\frac{\mu Q}{m_\phi^4}\right)^{1/3}, \\
&\omega \sim \mu \sim \frac{m_\phi^2}{\varphi_c}, \\
&\varphi_c \sim \left(\frac{m_\phi^2 M_P^{n-3}}{\lambda}\right)^{1/(n-1)}.
\end{aligned} \tag{10}$$

To see the transition region as well, we solve numerically Eq.(5) for the potential (9). In Fig. 2, we show the profile of the Q balls, where one can see the growth of the thin-wall-type Q ball as well as the deformation from the gauge-type Q balls as the charge is accumulating. Q-ball properties are shown in Fig. 3. They coincide to analytical estimates (7) and (10) for the gauge-type and thin-wall-type Q balls, respectively, and they are smoothly connected in between.

V. CONCLUSIONS

We have revisited the new-type of the Q ball, and clarified its properties. In particular, we have focused on the energy per charge E/Q as a function of the charge Q, and reconfirmed the stability of the new-type Q ball to be the

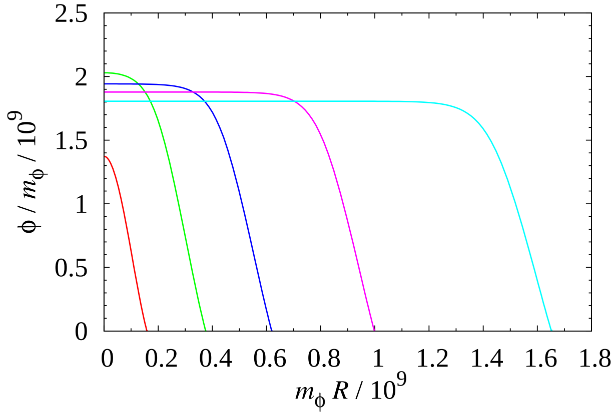

FIG. 2: Profile of the Q balls for $\omega/m_\phi = 2\times10^{-8}$, 1×10^{-8}, 8×10^{-9}, 7×10^{-9}, and 6×10^{-9} from the left to the right. They correspond to $Q = 9.4\times10^{34}$, 2.5×10^{36}, 1.3×10^{37}, 6.0×10^{37}, and 2.6×10^{38}, respectively.

dark matter of the universe. This corrects some misunderstandings in Ref. [13]. In addition, we have investigated the feature of the Q ball, transiting from the gauge-type to thin-wall-type Q balls, which was considered in the context of the Q-ball capture by the neutron star in Ref. [10]. Since the energy per charge E/Q decreases as the charge Q increases through the transition region, we confirm the 'metamorphosis' of the gauge-type to thin-wall-type Q balls.

Acknowledgments

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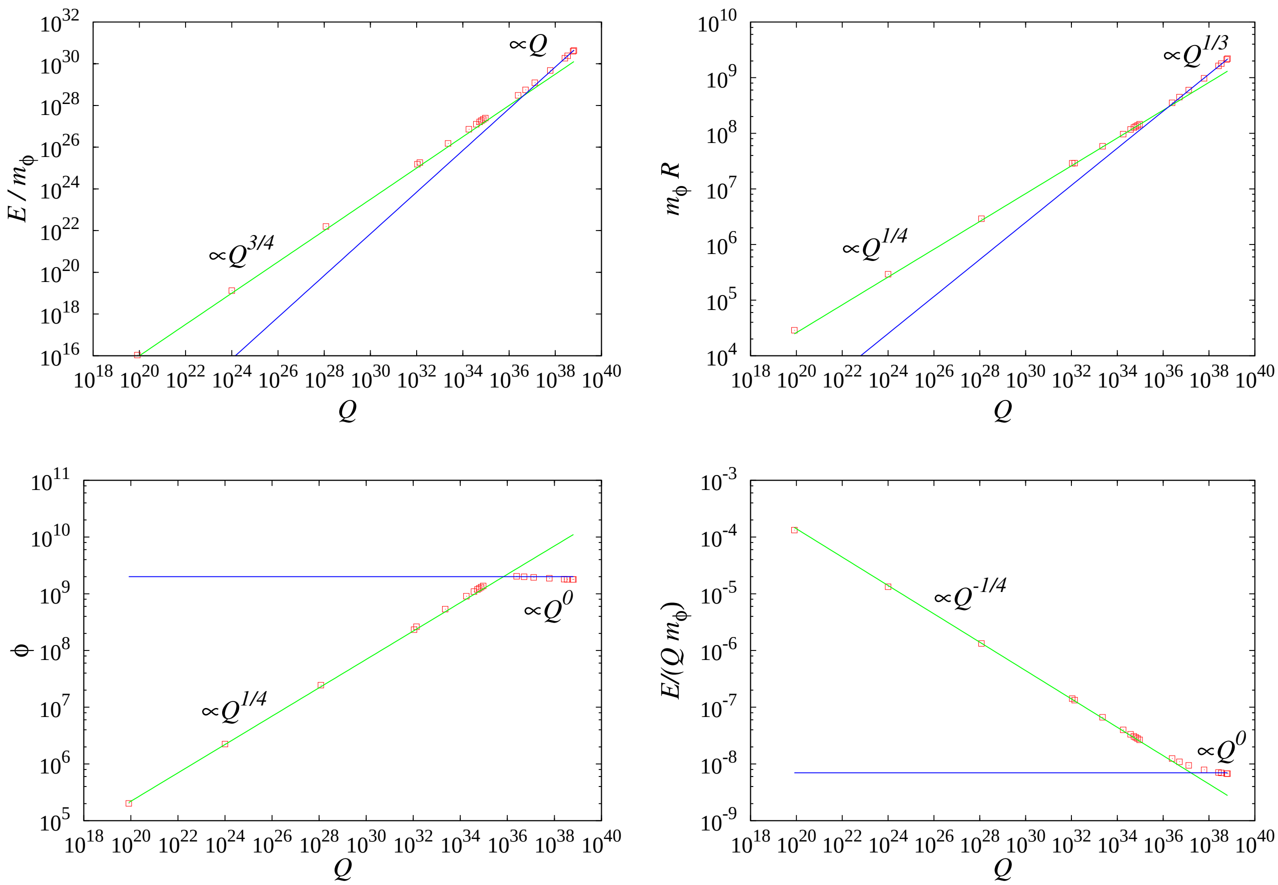

FIG. 3: Energy, size, field value at the center, and energy per charge of the Q balls. Green and blue lines show the Q-dependence estimated analytically for the gauge-type and thin-wall-type Q balls, respectively.